Enhancing Coverage and Capacity for Multiuser MIMO Systems by Utilizing Scheduling

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***Abstract*— Recent studies have revealed that the remarkable capacity improvement resulting from an open-loop multiple-input-multiple-output (MIMO) spatial multiplexing system may come at the sacrifice of degrading link reliability. This tradeoff between antenna multiplexing gain against antenna diversity gain may translate into smaller coverage areas. In this paper, we suggest using the multiuser diversity to replenish the diversity-deficient spatial multiplexing MIMO system. Specifically, we propose a fair scheduling scheme, called the strongest-weakest-normalized-subchannel-first (SWNSF) scheduling, which requires only limited amount of feedback. Our analysis and results indicate that the SWNSF scheduling can significantly increase the coverage of the multiuser MIMO system while further improving the system capacity.**

***Index Terms*— Spatial multiplexing, scheduling, fading channels, MIMO systems.**

I. INTRODUCTION

USING the spatial multiplexing technique to transmit signals over a point-to-point multiple-input-multiple-output (MIMO) channel has attracted great attentions because of its capability to deliver remarkable capacity gain [1]-[3]. However, recent studies have revealed that the large capacity benefit resulting from the spatial multiplexing MIMO system may come at the price of link reliability degradation when the prior channel information is not available at the transmitter [4]-[5]. In [4], the authors derived an optimal tradeoff curve for the maximum achievable diversity gain of an open-loop MIMO system given any realized multiplexing gain. In [5], the authors quantitatively evaluated some practical diversity-based and multiplexing-based MIMO schemes. Their numerical results indicated that it is difficult to simultaneously accomplish both diversity gain and multiplexing gain in an open-loop MIMO system. Due to the tradeoff of antenna multiplexing gain against antenna diversity gain, applying the spatial multiplexing scheme to transmit data over the MIMO channel may lead to smaller coverage areas subject to the same total transmit power and link reliability requirement. How to pursue high throughput with the spatial multiplexing MIMO scheme while maintaining satisfactory link reliability remains an open research issue.

In a multiuser system, on the other hand, multiuser diversity can be exploited to improve spectral efficiency by using the scheduling technique [6]-[7]. A number of works have also studied various scheduling techniques to enhance the uplink capacity [8]-[9] and the downlink capacity [10]-[21] for the multiuser MIMO system. According to whether the multiple users can be simultaneously served by the base station, the downlink scheduling scheme can be further categorized as follows:

- When the base station is permitted to serve only one user at a time, i.e. adopt the time division multiple access (TDMA) protocol, scheduling can be performed in the temporal domain to offer selection diversity gain by taking advantage of independent fading statistics across user population. Usually only a low-rate feedback channel is required to implement the temporal domain scheduling. Current wireless systems such as IS-856 [24] support low-rate feedback channels in the reverse link and adopt the TDMA protocol for downlink scheduling. Some works such as [10]-[15] have investigated the capacity benefit of applying the temporal domain scheduling to the multiuser MIMO system.
- By comparison, when the base station is allowed to simultaneously transmit multiple beams to different users, both the temporal and spatial (antenna) domains can be exploited by the scheduling to provide higher diversity order [16]-[17]. Because the signals to be transmitted for the multiple users are interfered with each other, the spatial-temporal domain scheduling is generally combined with additional pre-transmit signal processing techniques such as dirty-paper coding to achieve better performance [18]-[20]. Moreover, the spatial-temporal domain scheduling usually requires much higher amounts of feedback in order to obtain the full channel knowledge of each MIMO link. In [21], a random beam scheme was proposed to approach the achievable capacity of the spatial-temporal domain scheduling with a significant reduction of feedback.

This paper focuses on the downlink scheduling scheme that uses the TDMA protocol and requires only low-rate feedback in the multiuser MIMO system.

Besides the insight of diversity-multiplexing tradeoff in a point-to-point MIMO link observed in [4], [5], recent theoretic studies on the fundamental capacity limit of the multiuser MIMO system also provide valuable edification for designing practical multiuser MIMO systems. Consider a MIMO

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Gaussian broadcast channel where the base station with m transmit antennas serves K users with n receive antennas each. In [22] and [23], it was shown that the sum-rate capacity of such a multiuser MIMO system under the TDMA protocol scales like:

$$C_{\text{TDMA}} \sim \min(m, n) \log\log K \ . \quad (1)$$

The implications from the scaling law (1) are twofold. Firstly, given a fixed m, the number of the receive antennas n is critical to support a linear growth of capacity. Thus, using the spatial multiplexing MIMO technique in the physical layer can provide significant capacity gain. Secondly, the multiuser diversity extracted by scheduling techniques can only add to limited capacity benefit due to the factor of $\log\log K$.

Based on the above observations from (1) and the fundamental diversity-multiplexing tradeoff characteristic from [4] and [5], we advocate the new concept of using the multiuser diversity: Given a multiuser MIMO system with low-rate feedback and TDMA scheduling, a better strategy would be to devote the physical antennas to gaining capacity with the spatial multiplexing MIMO scheme and to focus the benefit of multiuser diversity on compensating the degraded link quality for the spatial multiplexing MIMO system. To our best knowledge, most scheduling algorithms in the literature were developed to improve capacity without noticing another degraded link reliability dimension of the MIMO system. In this work, we propose a scheduling scheme, called the strongest-weakest-normalized-subchannel-first (SWNSF) scheduling, to replenish the diversity-deficient spatial multiplexing MIMO system with multiuser diversity. The goal of the SWNSF scheduling scheme is to enhance the link reliability while further improving the link capacity. The SWNSF scheduling is a fair algorithm in the sense that users at different near-far locations have the same probability to receive services. Furthermore, this scheduling scheme requires only scalar feedback, meaning that each user only needs to send back a scalar value to indicate the channel condition, irrespective of the number of transmit and receive antennas. Therefore, the SWNSF scheduling algorithm can be applicable to the multiuser MIMO system with low-rate feedback channels and TDMA scheduling protocol. Through tractable eigenvalue analysis, we show that the SWNSF scheduling can greatly improve both the link reliability (in terms of coverage) and capacity of the multiuser MIMO system.

The rest of this paper is organized as follows. In Section II, we describe the channel model. In Section III, we introduce the SWNSF scheduling algorithm and present its key properties. In Section IV, we analyze the effect of the SWNSF scheduling on the coverage area. In Section V, by analysis, we show that the SWNSF scheduling can also increase capacity. In Section VI, we provide our concluding remarks.

II. Channel Model

As shown in Fig. 1, we consider a wireless system where a base station with m transmit antennas serves K downlink users each of which is equipped with n receive antennas. We assume that the spatial multiplexing method is used for data transmission between the base station and any target user. For clarifying the effect of multiuser scheduling on the supplement of diversity gain, we assume $m = n$ since in this case all the degrees of freedom in the MIMO system are exhausted for multiplexing gain [4]. Thus, the link between the base station and each individual user constitutes an $n \times n$ MIMO system. Let $\mathbf{x}_k$ and $\mathbf{y}_k$ be the $n \times 1$ transmit and receive signal vectors of user k, respectively, $\mathbf{G}_k$ the $n \times n$ channel matrix between the base station and user k, and $\mathbf{n}_k$ the $n \times 1$ spatially white noise vector with $\mathrm{E}[\mathbf{n}_k \mathbf{n}_k^\dagger] = \sigma_n^2 \mathbf{I}$, where $(\cdot)^\dagger$ is the transpose conjugate operation. Thus, the link between $\mathbf{x}_k$ and $\mathbf{y}_k$ is related by [25]

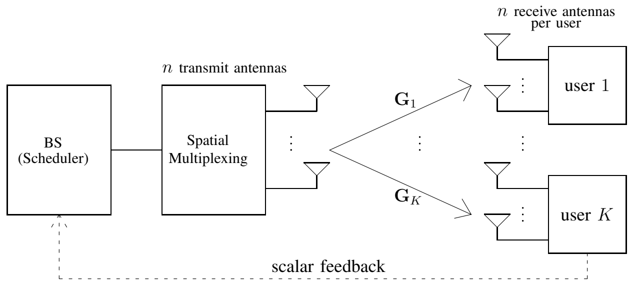

Fig. 1. A MIMO multiuser scheduling system.

$$\mathbf{y}_k = \mathbf{G}_k \mathbf{x}_k + \mathbf{n_k} = \sqrt{g_k}\, \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k \ , \quad (2)$$

where g_k depicts the large-scale (local average) behavior of the channel gain, and $\mathbf{H}_k$ captures the channel fading characteristics. For user k at a distance of r_k from the base station, g_k can be generally represented by [26]

$$10 \log_{10}(g_k) = -10\mu \log_{10}(r_k) + g_0 \ \ [\text{dB}] \ , \quad (3)$$

where μ is the path loss exponent and g_0 is a system dependent constant. For the $n \times n$ normalized channel matrix $\mathbf{H}_k$, we assume that every entries of $\mathbf{H}_k$ are pairwise independent, circular-symmetric complex Gaussian random variable $\mathcal{CN}(0,1)$, meaning that the real and imaginary parts of each entry have uncorrelated Gaussian distributions with zero mean and variance $1/2$. The total transmit power at the base station is constrained by P_t, i.e. $\mathrm{E}[\mathbf{x}_k^\dagger \mathbf{x}_k] \le P_t$.

With the link model defined in (2), the ergodic capacity (nats/sec/Hz) of an open-loop MIMO system under TDMA operation with equal power allocation among transmit antennas is given by [1]

$$C_k = \mathrm{E}\left[\log\det\left(I + \frac{\rho_k}{n} \mathbf{H}_k \mathbf{H}_k^\dagger \right) \right] \quad (4)$$

where

$$\rho_k = P_t\, g_k / \sigma_n^2 = (P_t/\sigma_n^2)\, r_k^{-\mu}\, 10^{g_0/10} \quad (5)$$

is the average receive signal-to-noise ratio (SNR) of user k. By applying the singular value decomposition (SVD) to $\mathbf{H}_k$ of (4), C_k can be also represented by

$$C_k = \sum_{i=1}^{n} \mathrm{E}\left[\log\left(1 + \frac{\rho_k}{n} \lambda_{k,i} \right) \right] \ , \quad (6)$$

where $\{\lambda_{k,i}\}_{i=1}^n$ are the eigenvalues of the Wishart matrix $\mathbf{H}_k \mathbf{H}_k^\dagger$ for user k. Without loss of generality, we arrange $\{\lambda_{k,i}\}_{i=1}^n$ in the decreasing order so that $\lambda_{k,1} \ge \lambda_{k,2} \ge$

$\cdots \geq \lambda_{k,n} \geq 0$[1]. Comparing to the ergodic capacity of the single-input-single-output (SISO) system, the ergodic capacity of an open-loop MIMO channel is enhanced by forming n parallel subchannels, each of which has effective output SNR $\gamma_{k,i} = \rho_k \lambda_{k,i}/n$ at the receive antenna. In this paper, we refer to the link capacity as the ergodic capacity of the information theory.

In addition to the link capacity, let's examine the link outage probability of the MIMO system because it reflects how reliable a MIMO channel can support the corresponding capacity. For the SISO system, a common definition of link outage is the probability that the receive SNR is less than a predetermined value γ_{th}, i.e. $P_{\text{out}} = \Pr\{\gamma < \gamma_{\text{th}}\}$ [27]. Based on the insight from [5], the link outage for the spatial multiplexing MIMO system can be generalized as follows. By noting that all the subchannels of the spatial multiplexing MIMO system are used for independent data transmissions in parallel, the link outage for the spatial multiplexing MIMO system can be defined as the event when the receive SNR of any subchannel is less than γ_{th}. That is, for any user k,

$$\begin{aligned} P_{\text{out}}^k &= \Pr\{\gamma_{k,1} < \gamma_{\text{th}} \cup \gamma_{k,2} < \gamma_{\text{th}} \cdots \cup \gamma_{k,n} < \gamma_{\text{th}}\} \\ &= 1 - \Pr\left\{\min_{\forall i}\{\gamma_{k,i}\} = \gamma_{k,n} \geq \gamma_{\text{th}}\right\} \\ &= \Pr\{\gamma_{k,n} < \gamma_{\text{th}}\} \ . \end{aligned} \quad (7)$$

Intuitively, the weakest subchannel with the lowest SNR in the MIMO system dominates the outage probability performance since the weakest subchannel is most likely to incur transmission errors. By writing (7) as $\Pr\{\gamma_{k,n} \geq \gamma_{\text{th}}\} = 1 - P_{\text{out}}^k$, (7) can be also interpreted as that, allowing for a certain probability $(1 - P_{\text{out}}^k)$, all receive SNR in subchannels of the MIMO system are required to be greater than γ_{th}. In Section IV, we will relate (7) to the resulting cell coverage.

III. Multiuser Scheduling

In this paper, we assume that multiple users are served by a base station in a time division multiple access manner. In each time slot, the base station selects a target user according to a certain scheduling policy. For any user k, the channel $\mathbf{H}_k$ is assumed to be fixed during a time slot, but independently varies between different time slots. The variation of $\mathbf{H}_k$ across users is also assumed to be independent. In the following, we introduce two scheduling algorithms. The first one is the traditional round-robin scheduling method, which serves as the baseline performance for comparison in this work. The second one is our proposed strongest-weakest-normalized-subchannel-first scheduling algorithm.

A. Round-Robin Scheduling

The round-robin (RR) scheduling assigns each time slot to a target user in the circularly sequential manner among K users. The round-robin scheduling is known for its fairness and low implementation cost. In a multiuser system where users can have different near-far locations, the RR scheduling provides a fair allocation scheme for numerous near-far users to access services with equal probability. However, the RR scheduling can not exploit multiuser diversity since it ignores the channel conditions among user population. Next, we study another fair scheduling algorithm that can take advantage of multiuser diversity to offer additional scheduling gain.

B. Strongest-Weakest-Normalized-Subchannel-First Scheduling

From a multiuser system point of view, the base station can see the total nK antennas at the receive end rather than n links. In a diversity-deficient spatial multiplexing MIMO system where all physical antennas are dedicated to capacity enhancement, the additional $(K-1)n$ virtual receive antennas from the other users can be exploited to improve reliability performance. Per the observation from (7), we introduce a scheduling algorithm to take advantage of multiuser diversity to directly cope with the weakest subchannel of the MIMO system. The proposed algorithm, called the strongest-weakest-normalized-subchannel-first (SWNSF) scheduling, selects the target user with the maximum $\lambda_{k,n}$ among K users at each time slot:

$$k^* = \arg\max_k \frac{\lambda_{\min}\left(\mathbf{G}_k \mathbf{G}_k^\dagger\right)}{g_k} \quad (8)$$

$$= \arg\max_k \lambda_{\min}\left(\mathbf{H}_k \mathbf{H}_k^\dagger\right)$$

$$= \arg\max_k \lambda_{k,n} \ . \quad (9)$$

where $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue. In (8), g_k in the denominator is used to equalize the average SNR among different users due to the near-far effect. In practice, g_k can be estimated based on the long-term measurements at the user side. We assume that each user can accurately estimate the value of $\lambda_{k,n}$ and correctly send it back to the base station without delay.

Let $\tilde{\mathbf{H}}_k$ and $\tilde{\gamma}_{k,n}$ be the normalized channel matrix and the output SNR of the weakest subchannel for user k determined by the SWNSF scheduling algorithm, respectively. Accordingly, $\tilde{\lambda}_{k,n} = \tilde{\gamma}_{k,n} n/\rho_k$ is the minimum eigenvalue of $\tilde{\mathbf{H}}_k \tilde{\mathbf{H}}_k^\dagger$. Thus, the following proposition summaries the important properties of the SWNSF scheduling algorithm.

Proposition 1: Assume that K users with different $\rho_k \, (1 \leq k \leq K)$ are served according to the SWNSF scheduling policy defined in (9). Let p_k be the probability of user k being served at any time slot. Then we have:

(i) The SWNSF scheduling is fair in the sense $p_k = 1/K$ for $1 \leq k \leq K$.

(ii) The cumulative distribution function (CDF) of $\tilde{\gamma}_{k,n}$ is $F_{\tilde{\gamma}_{k,n}}(\gamma) = \left[F_{\gamma_{k,n}}(\gamma)\right]^K$ for any k, where $F_{\gamma_{k,n}}(\gamma)$ is the CDF of $\gamma_{k,n}$.

Proof: From (9), any user k competes for services with the other $(K-1)$ users at each time slot. Because $\{\lambda_{k,n}\}_{k=1}^K$ are independent and identically distributed (i.i.d.) random variables, we have $p_k = 1/K$. Furthermore, the CDF of $\tilde{\lambda}_{k,n}$ is given by

$$F_{\tilde{\lambda}_{k,n}}(\lambda) = \Pr\left\{\max_k(\lambda_{k,n}) < \lambda\right\} = \left[F_{\lambda_{k,n}}(\lambda)\right]^K \ . \quad (10)$$

[1]It is shown in [28] that the random Wishart matrix $\mathbf{H}_k \mathbf{H}_k^\dagger$ has n distinct eigenvalues with probability one.

Since $\tilde{\gamma}_{k,n} = \rho_k \tilde{\lambda}_{k,n}/n$ and $\gamma_{k,n} = \rho_k \lambda_{k,n}/n$, we can have

$$F_{\tilde{\gamma}_{k,n}}(\gamma) = F_{\tilde{\lambda}_{k,n}}\left(\frac{n\gamma}{\rho_k}\right) = \left[F_{\lambda_{k,n}}\left(\frac{n\gamma}{\rho_k}\right)\right]^K = \left[F_{\gamma_{k,n}}(\gamma)\right]^K . \quad (11)$$

□

Proposition 1 states that the SWNSF scheduling can capture the variations of independent channel fading to achieve multiuser diversity, while maintaining fairness among users. Next, we investigate how the RR and SWNSF scheduling algorithms affect the coverage and capacity of the multiuser MIMO system.

IV. Effect of SWNSF Scheduling on Coverage

In this section, we investigate the effect of multiuser scheduling on cell coverage. We consider a single cell with the background noise. Referring to (7) and the path loss related link budget (5), we shall determine the cell radius by the maximum distance at which the link quality suffices for maintaining a required receive SNR γ_{th} with the probability at least $(1 - P_{\text{out}})$ [30], [31]. Since the user at the cell boundary is the major concern in determining cell coverage, we may omit the user index k to ease notations in this section.

A. Characteristics of $\lambda_{k,n}$ in the MIMO system

Now we introduce Proposition 2 to characterize the statistical distribution of $\lambda_{k,n}$.

Proposition 2: Let $\lambda_{k,1} \geq \lambda_{k,2} \geq \cdots \geq \lambda_{k,n} \geq 0$ be the *ordered eigenvalues* of the Wishart matrix $\mathbf{H}_k\mathbf{H}_k^\dagger$, where $\mathbf{H}_k$ is an $n \times n$ channel matrix with each entry $h_{ij} \sim \mathcal{CN}(0,1)$. Then the marginal probability density function (PDF) of the minimum eigenvalue $\lambda_{k,n}$ is exponentially distributed with parameter n [33].

Proof: From [32], for an $n \times m$ $(n \geq m)$ channel matrix $\mathbf{H}_k$, the joint distribution function of the ordered eigenvalues of $\mathbf{H}_k\mathbf{H}_k^\dagger$ is

$$f_{\lambda_{k,1},\cdots,\lambda_{k,n}}(\lambda_1,\cdots,\lambda_n) = \frac{\pi^{n(n-1)}}{\Gamma_m(n)\Gamma_n(m)} \cdot \exp\left(-\sum_{i=1}^{n}\lambda_i\right)\prod_{i=1}^{n}\lambda_i^{m-n}\prod_{i<j}(\lambda_i - \lambda_j)^2 , \quad (12)$$

where $\Gamma_a(x)$ is the complex multivariate gamma function defined by

$$\Gamma_a(x) = \pi^{a(a-1)/2}\prod_{i=1}^{a}\Gamma(x - i + 1) . \quad (13)$$

Note that $\Gamma(z) = \int_0^\infty t^{z-1}e^{-t}dt$ is the gamma function in (13). In the case of $m = n$, (12) is reduced to

$$f_{\lambda_{k,1},\cdots,\lambda_{k,n}}(\lambda_1,\cdots,\lambda_n) = \frac{1}{\left[\prod_{i=1}^{n}\Gamma(i)\right]^2}\exp\left(-\sum_{i=1}^{n}\lambda_i\right)\prod_{i<j}(\lambda_i - \lambda_j)^2 . \quad (14)$$

Therefore, the marginal distribution of $\lambda_{k,n}$ can be obtained by integrating over all the other variables as follows:

$$f_{\lambda_{k,n}}(\lambda_n) = \frac{1}{\left[\prod_{i=1}^{n}\Gamma(i)\right]^2}\int_{\lambda_n}^{\infty}\int_{\lambda_{n-1}}^{\infty}\cdots\int_{\lambda_2}^{\infty} \exp\left(-\sum_{i=1}^{n}\lambda_i\right)\prod_{i<j}(\lambda_i - \lambda_j)^2\, d\lambda_1\cdots d\lambda_{n-2}d\lambda_{n-1} . \quad (15)$$

By making the transformation $x_i = \lambda_i - \lambda_n$ for $i = 1,\cdots,n-1$, (15) can be written as

$$f_{\lambda_{k,n}}(\lambda_n) = e^{-n\lambda_n}\left\{\frac{1}{\left[\prod_{i=1}^{n}\Gamma(i)\right]^2}\int_{0}^{\infty}\int_{x_{n-1}}^{\infty}\cdots\int_{x_2}^{\infty} \exp\left(-\sum_{i=1}^{n-1}x_i\right)\prod_{i=1}^{n-1}x_i^2\cdot\prod_{\substack{i<j\\ i,j\in\{1,\cdots,n-1\}}}(x_i - x_j)^2\, dx_1\cdots dx_{n-2}dx_{n-1}\right\} . \quad (16)$$

Since the $(n-1)$ fold integrals in the bracket of (16) involve only variables $x_1,\cdots,x_{n-1}$ and $f_{\lambda_{k,n}}(\lambda_n)$ is a probability density function, we can have

$$f_{\lambda_{k,n}}(\lambda_n) = ne^{-n\lambda_n},\ \ \lambda_n \geq 0 . \quad (17)$$

□

B. Cell Radius with RR Scheduling

Since the RR scheduling selects the target user in a statistically random manner, the receive SNR of the target user will not be affected by the RR scheduling algorithm. Thus, according to Proposition 2 and $\gamma_{k,n} = \rho_k\lambda_{k,n}/n$, the receive SNR $\gamma_{k,n}$ of any target user under the RR scheduling is an exponentially distributed random variable with the following CDF

$$F_{\gamma_{k,n}}(\gamma) = 1 - \exp\left(-\frac{n^2\gamma}{\rho_k}\right),\ \ \gamma \geq 0 . \quad (18)$$

Suppose that the cell radius is defined by the farthest distance at which the link quality suffices for maintaining a required receive SNR γ_{th} with the probability no less than $(1 - P_{\text{out}})$. Thus, by substituting (18) and (5) into (7), the cell radius of the multiuser MIMO system employing the RR scheduling is given by

$$r = \left[\left(\frac{P_t}{\sigma_n^2}\right)\left(\frac{10^{g_0/10}}{n^2\,\gamma_{\text{th}}}\right)\log\left(\frac{1}{1 - P_{\text{out}}}\right)\right]^{1/\mu} . \quad (19)$$

For $n = 1$ or the SISO case, the cell radius of (19) becomes

$$r_{\text{SISO}} = \left[\left(\frac{P_t}{\sigma_n^2}\right)\left(\frac{10^{g_0/10}}{\gamma_{\text{th}}}\right)\log\left(\frac{1}{1 - P_{\text{out}}}\right)\right]^{1/\mu} . \quad (20)$$

Comparing (19) and (20) under the same total transmit power of a base station P_t and the same link reliability requirements P_{out} and γ_{th}, we have

$$r = \left(\frac{1}{n^2}\right)^{1/\mu} r_{\text{SISO}} . \quad (21)$$

As a result, the cell radius of the spatial multiplexing multiuser MIMO system shrinks under the RR scheduling. Next, we derive the cell radius of the multiuser MIMO system using the SWNSF scheduling.

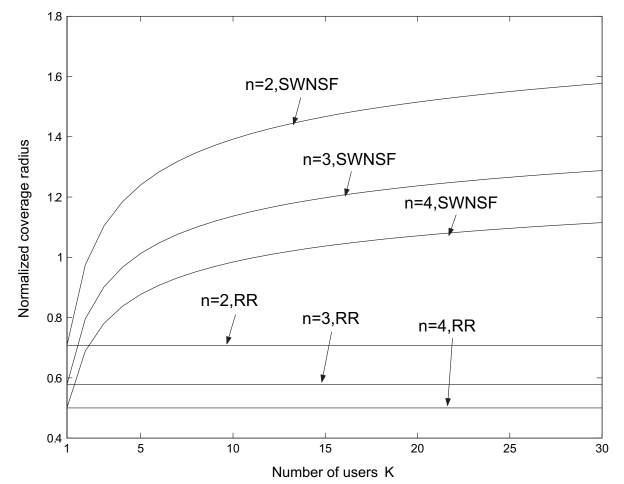

Fig. 2. Comparison of the cell radius of the multiuser MIMO system using the RR and SWNSF scheduling.

C. Cell Radius with SWNSF Scheduling

When the connection between the base station and the target user is established by the SWNSF scheduling, the receive SNR $\tilde{\gamma}_{k,n}$ is changed according to (11). By substituting (18) into (11), the CDF of $\tilde{\gamma}_{k,n}$ subject to the SWNSF scheduling becomes

$$F_{\tilde{\gamma}_{k,n}}(\gamma) = \left[1 - \exp\left(-\frac{n^2\gamma}{\rho_k}\right)\right]^K , \quad \gamma \geq 0 \ . \tag{22}$$

Analogous to (19), applying (22) and (5) to (7), we can obtain the cell radius of the multiuser MIMO system with the SWNSF scheduling as follows:

$$\tilde{r} = \left[\left(\frac{P_t}{\sigma_n^2}\right)\left(\frac{10^{g_0/10}}{n^2\,\gamma_{\text{th}}}\right)\log\left(\frac{1}{1-\sqrt[K]{P_{\text{out}}}}\right)\right]^{1/\mu} . \tag{23}$$

We can also compare $\tilde{r}$ in (23) and r_{SISO} in (20) to have

$$\tilde{r} = \left[\frac{1}{n^2}\log\left(\frac{1}{1-\sqrt[K]{P_{\text{out}}}}\right)\Big/\log\left(\frac{1}{1-P_{\text{out}}}\right)\right]^{1/\mu} r_{\text{SISO}}$$
$$\simeq \left[\frac{1}{n^2}\left(\frac{1}{P_{\text{out}}}-\frac{1}{2}\right)\log\left(\frac{1}{1-\sqrt[K]{P_{\text{out}}}}\right)\right]^{1/\mu} r_{\text{SISO}}, \quad \text{for small } P_{\text{out}} \ . \tag{24}$$

Unlike (21) where r is always smaller than r_{SISO}, $\tilde{r}$ in (24) can be possibly greater than r_{SISO} with a sufficient number of users in the system, i.e.

$$K \geq \left\lceil \frac{\log(P_{\text{out}})}{\log\left(1-\exp\left(-\frac{2n^2P_{\text{out}}}{2-P_{\text{out}}}\right)\right)} \right\rceil , \tag{25}$$

where $\lceil x \rceil$ denotes the smallest integer greater or equal to x. Next, we give a numerical example to compare the resulting cell radius when adopting the RR and SWNSF scheduling policies.

D. Numerical Example

Figure 2 plots the cell radius of the multiuser MIMO system using the RR and SWNSF scheduling according to (21) and (24). We normalize r_{SISO} to unity and set the path loss exponent $\mu = 4$ in this example. Also, we require the link reliability of the user at the cell boundary to be adequately covered 90% of the time, i.e. $P_{\text{out}} = 0.1$. From Fig. 2, it is shown that the cell radius of the spatial multiplexing MIMO system with the RR scheduling reduces as compared with SISO case. However, applying the SWNSF scheduling can enhance the receive SNR, thereby extending the cell radius. When there are sufficiently many users in the system, e.g. $K = 5$ for $n = 3$ and $K = 12$ for $n = 4$, the cell radius with the SWNSF scheduling is greater than r_{SISO}.

V. Effect of SWNSF Scheduling on Capacity

In the previous section, we have demonstrated that the SWNSF scheduling can extend the coverage of the multiuser MIMO system. Now, we proceed to investigate the capacity benefit resulting from the SWNSF scheduling. In order to evaluate the capacity improvement resulting from the SWNSF scheduling, we need to characterize the statistical behavior of all the eigenvalues $\{\tilde{\lambda}_{k,i}\}_{i=1}^{n}$. In what follows, we first derive the mean increment of $(\tilde{\lambda}_{k,n} - \lambda_{k,n})$ due to the SWNSF scheduling. Then, we examine the mean increment of $(\tilde{\lambda}_{k,i} - \lambda_{k,i})$ of all the other ordered eigenvalues $\{\tilde{\lambda}_{k,i}\}_{i=1}^{n-1}$. Finally, we derive the link and system capacity for the multiuser MIMO system employing the SWNSF scheduling.

A. Analysis of $\tilde{\lambda}_{k,n}$

Recall from Proposition 2 that $\lambda_{k,n}$ is an exponentially distributed random variable with parameter n. From (10), the SWNSF scheduling has changed the CDF of the minimum eigenvalue into

$$F_{\tilde{\lambda}_{k,n}}(\lambda) = \left(1 - e^{-n\lambda}\right)^K \ . \tag{26}$$

The mean of $\tilde{\lambda}_{k,n}$ can be calculated as follows

$$\begin{aligned} \mathrm{E}[\tilde{\lambda}_{k,n}] &\overset{(a)}{=} \int_0^\infty (1 - F_{\tilde{\lambda}_{k,n}}(\lambda))d\lambda \\ &\overset{(b)}{=} \int_0^\infty \sum_{k=1}^{K}\binom{K}{k}(-1)^{k+1}e^{-nk\lambda}d\lambda \\ &= \frac{1}{n}\sum_{k=1}^{K}\binom{K}{k}\frac{(-1)^{k+1}}{k} \\ &\overset{(c)}{=} \frac{1}{n}\sum_{k=1}^{K}\frac{1}{k} \end{aligned} \tag{27}$$

where (a) relies on $\tilde{\lambda}_{k,n} \geq 0$ [38], (b) uses the binomial expansion and (c) follows from [34, eq. 0.155]. Equivalently, the mean increment of $\tilde{\lambda}_{k,n}$ relative to $\lambda_{k,n}$ can be expressed by

$$\Delta_\lambda \triangleq \mathrm{E}[\,\tilde{\lambda}_{k,n}\,] - \mathrm{E}[\,\lambda_{k,n}\,] = \frac{1}{n}\left[\psi(K+1) + \beta - 1\right] \ , \tag{28}$$

where $\psi(K+1) = -\beta + \sum_{k=1}^{K} k^{-1}$ is the psi function for integer K and $\beta \simeq 0.5772$ is the Euler's constant [29]. We note that the psi function $\psi(K+1)$ can be approximated by $\log(K+1) \simeq \log K$ for sufficiently large K [29, eq. 6.3.18].

B. Analysis of $\{\tilde{\lambda}_{k,i}\}_{i=1}^{n-1}$

With the statistical knowledge of $\tilde{\lambda}_{k,n}$, we proceed to investigate the behavior of $\{\tilde{\lambda}_{k,i}\}_{i=1}^{n-1}$ when applying the SWNSF algorithm. We introduce a new set of random variables as follows:

$$\begin{aligned} s_{k,n} &\triangleq \lambda_{k,n} \\ s_{k,i} &\triangleq \lambda_{k,i} - \lambda_{k,n}, \quad \text{for } 1 \le i \le n-1 \ . \end{aligned} \quad (29)$$

Obviously, $s_{k,i}$ denote the "spacing" between $\lambda_{k,i}$ and the smallest eigenvalue $\lambda_{k,n}$ for $1 \le i \le n-1$. Through $s_{k,i}$, the following proposition describes some statistical characteristics of $\tilde{\lambda}_{k,i}$.

Proposition 3: Let $s_{k,i}$ be the random variables defined in (29). Let $\phi_{\tilde{\lambda}_{k,i}}(\omega)$, $\phi_{\lambda_{k,i}}(\omega)$ and $\phi_{s_{k,i}}(\omega)$ denote the Laplace transform[2] of the PDFs of $\tilde{\lambda}_{k,i}$, $\lambda_{k,i}$ and $s_{k,i}$, respectively. Then we have

(i) The spacing $s_{k,i}$ are independent of $\lambda_{k,n}$ for $1 \le i \le n-1$.

(ii) $\phi_{\tilde{\lambda}_{k,i}}(\omega)$, $\phi_{\lambda_{k,i}}(\omega)$ and $\phi_{s_{k,i}}(\omega)$ are related by

$$\phi_{\tilde{\lambda}_{k,i}}(\omega) = \phi_{\tilde{\lambda}_{k,n}}(\omega)\phi_{s_{k,i}}(\omega) = A(\omega)\phi_{\lambda_{k,i}}(\omega) \ , \quad (30)$$

where $A(\omega) = \frac{K!\,(1+\omega/n)\,\Gamma(1+\omega/n)}{\Gamma(1+K+\omega/n)}$, $\Gamma(\cdot)$ is the gamma function and $K! = \prod_{i=1}^{K} i$.

(iii) The mean increment between $\tilde{\lambda}_{k,i}$ and $\lambda_{k,i}$ is

$$\begin{aligned} \mathrm{E}[\tilde{\lambda}_{k,i}] - \mathrm{E}[\lambda_{k,i}] &= \Delta_\lambda \\ &= \frac{1}{n}\left[\psi(K+1) + \beta - 1\right], \quad 1 \le i \le n-1 \ . \end{aligned} \quad (31)$$

(iv) The sum of $\mathrm{E}[\tilde{\lambda}_{k,i}]$ over all eigenvalues is

$$\sum_{i=1}^{n} \mathrm{E}[\tilde{\lambda}_{k,i}] = n^2 + n\,\Delta_\lambda \ . \quad (32)$$

Proof: Please refer to Appendix A. □

By comparing (28) and (31), it is interesting to find that the mean increment $\Delta_\lambda = \mathrm{E}[\,\tilde{\lambda}_{k,n} - \lambda_{k,n}\,]$ resulting from the SWNSF scheduling has been shifted to all the other ordered eigenvalues. In other words, the SWNSF scheduling can increase the value of $\mathrm{E}[\tilde{\lambda}_{k,n}]$ and subsequently the *repelling property* between $\tilde{\lambda}_{k,i}$ and $\tilde{\lambda}_{k,n}$ (as implied by Proposition 3) brings about the same amount of enhancement for $\mathrm{E}[\tilde{\lambda}_{k,i}]$. The effect of the SWNSF scheduling on the movement of $\lambda_{k,i}$ can be best illustrated by the following example.

Example 1: Consider a 3×3 MIMO system. Following (12), the joint PDF of the ordered eigenvalues of $\mathbf{H}_k\mathbf{H}_k^\dagger$ is given as

$$\begin{aligned} f_{\lambda_{k,1},\lambda_{k,2},\lambda_{k,3}}(\lambda_1,\lambda_2,\lambda_3) &= \frac{1}{4}\,e^{-(\lambda_1+\lambda_2+\lambda_3)}(\lambda_1-\lambda_2)^2 \\ &\cdot(\lambda_1-\lambda_3)^2(\lambda_2-\lambda_3)^2 \ , \ \lambda_1 \ge \lambda_2 \ge \lambda_3 \ge 0 \ . \end{aligned} \quad (33)$$

Then the marginal CDF associated with $\lambda_{k,3}$, $\lambda_{k,2}$ and $\lambda_{k,1}$ can be respectively derived as

$$\begin{aligned} F_{\lambda_{k,3}}(\lambda) &= \int_0^{\lambda}\int_{\lambda_3}^{\infty}\int_{\lambda_2}^{\infty} f_{\lambda_{k,1},\lambda_{k,2},\lambda_{k,3}}(\lambda_1,\lambda_2,\lambda_3)d\lambda_1 d\lambda_2 d\lambda_3 \\ &=1 - e^{-3\lambda} \ , \end{aligned} \quad (34)$$

[2]The Laplace transform of the PDF of a continuous nonnegative random variable X is defined by $\phi_X(w) = \int_0^\infty e^{-\omega x} f_X(x)dx$ [29].

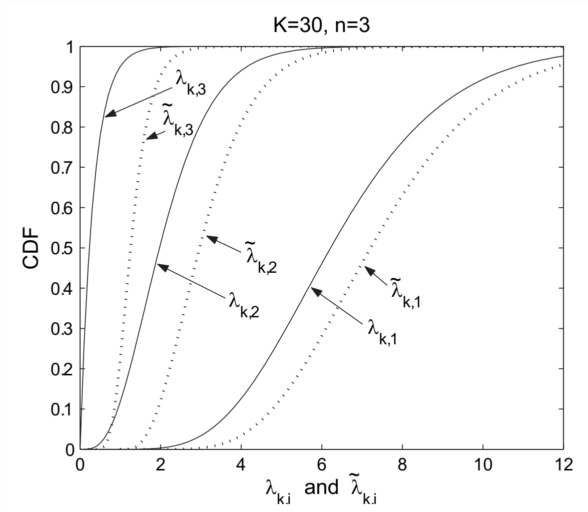

Fig. 3. The CDFs of $\lambda_{k,i}$ and $\tilde{\lambda}_{k,i}$ for $K = 30$ and $n = 3$.

$$\begin{aligned} F_{\lambda_{k,2}}(\lambda) &= \int_0^{\lambda}\int_0^{\lambda_2}\int_{\lambda_2}^{\infty} f_{\lambda_{k,1},\lambda_{k,2},\lambda_{k,3}}(\lambda_1,\lambda_2,\lambda_3)d\lambda_1 d\lambda_3 d\lambda_2 \\ &=1 + 2e^{-3\lambda} - e^{-2\lambda}\left(3 + 3\lambda^2 + \lambda^3 + \frac{\lambda^4}{4}\right) \ , \end{aligned} \quad (35)$$

and

$$\begin{aligned} F_{\lambda_{k,1}}(\lambda) &= \int_0^{\lambda}\int_0^{\lambda_1}\int_0^{\lambda_2} f_{\lambda_{k,1},\lambda_{k,2},\lambda_{k,3}}(\lambda_1,\lambda_2,\lambda_3)d\lambda_3 d\lambda_2 d\lambda_1 \\ &=1 - e^{-3\lambda} + e^{-2\lambda}\left(3 + 3\lambda^2 + \lambda^3 + \frac{\lambda^4}{4}\right) \\ &\quad - e^{-\lambda}\left(3 + 3\lambda^2 - \lambda^3 + \frac{\lambda^4}{4}\right) \ . \end{aligned} \quad (36)$$

On the other hand, from (26), the CDF of the smallest eigenvalue with the effect of the SWNSF scheduling becomes

$$F_{\tilde{\lambda}_{k,3}}(\lambda) = \left(1 - e^{-3\lambda}\right)^K \ . \quad (37)$$

By using (35) to obtain the Laplace transform of $f_{\lambda_{k,2}}(\lambda)$ and invoking (30), the Laplace transform of $f_{\tilde{\lambda}_{k,2}}(\lambda)$ is given as

$$\phi_{\tilde{\lambda}_{k,2}}(\omega) = \frac{K!\,(32 + 22\omega + 4\omega^2)\,\Gamma(1+\omega/3)}{\Gamma(1+K+\omega/3)(2+\omega)^5} \ . \quad (38)$$

Similarly, we can have

$$\phi_{\tilde{\lambda}_{k,1}}(\omega) = \frac{K!\,(32 + 42\omega + 14\omega^2)\,\Gamma(1+\omega/3)}{\Gamma(1+K+\omega/3)(2+3\omega+\omega^2)^5} \ . \quad (39)$$

Then the PDFs and CDFs of $\tilde{\lambda}_{k,2}$ and $\tilde{\lambda}_{k,1}$ can be obtained from (38) and (39) by using the numerical inversion [35].

Figure 3 plots the CDFs of all the eigenvalues with the effect of the SWNSF scheduling for $K = 30$ and $n = 3$. Comparing $F_{\lambda_{k,3}}(\lambda)$ and $F_{\tilde{\lambda}_{k,3}}(\lambda)$, one can see that the SWNSF scheduling causes the CDF of the smallest eigenvalue to move rightward, implying that the link reliability between the base station and the target user is improved. Furthermore, the CDFs of all the other ordered eigenvalues are repelled to move rightward accordingly. Essentially, the property of the rightward shift for all $F_{\lambda_{k,i}}(\lambda)$ enables further capacity improvements for the spatial multiplexing MIMO system. ◇

C. Scheduling Gain for Mean SNR Improvement

From (32) and (53), we can further write

$$\frac{\sum_{i=1}^{n}\mathrm{E}[\tilde{\lambda}_{k,i}]}{\sum_{i=1}^{n}\mathrm{E}[\lambda_{k,i}]} = \frac{\sum_{i=1}^{n}\mathrm{E}[\tilde{\gamma}_{k,i}]}{\sum_{i=1}^{n}\mathrm{E}[\gamma_{k,i}]} = 1 + \frac{1}{n^2}\left[\psi(K+1)+\beta-1\right] \quad . \tag{40}$$

The expression (40) indicates that the sum of the mean receive SNR at all subchannels of the spatial multiplexing MIMO system is improved thanks to the SWNSF scheduling. Based on (40), we can make the comments regarding the SWNSF scheduling gain as follows:

- The SWNSF scheduling can improve the sum of receive subchannel SNR for the MIMO system by exploiting multiuser diversity. When user population is large, the improvement of the mean receive SNR due to the SWNSF scheduling grows with $\log(K)/n^2$.
- The large number of antenna elements could attenuate the multiuser diversity gain resulting from the SWNSF scheduling. This phenomenon can be intuitively explained as follows. Generally speaking, scheduling is a media access layer (MAC) technique to deliver multiuser diversity gain by utilizing independent channel fluctuations among users. Therefore, the larger the channel variations among users, the higher the performance gain from scheduling. In this regard, it is shown in (17) that the number of antennas reduces the variance of $\lambda_{k,n}$ to $1/n^2$. Thus, the damped channel fluctuation reduces the scope that the SWNSF scheduling can exploit, thereby diluting the scheduling gain. A recent work [36] also observed the phenomenon of diminishing scheduling gain and called it the "channel-hardening" effect. Fortunately, with the antenna number of practical interests, the SWNSF scheduling can still provide considerable scheduling gain.

D. Capacity Analysis

Now we proceed to derive the link capacity and system capacity with the SWNSF scheduling. We define the system capacity as the sum of the link capacity delivered to each user on average. Therefore, the system capacity with the SWNSF scheduling can be written as

$$\langle C\rangle_{\text{SWNSF}} = \sum_{k=1}^{K} p_k \tilde{C}_k = \frac{1}{K}\sum_{k=1}^{K}\tilde{C}_k \quad , \tag{41}$$

where $p_k = 1/K$ follows from Proposition 1 and

$$\tilde{C}_k = \sum_{i=1}^{n}\mathrm{E}\left[\log(1+\tilde{\gamma}_{k,i})\right] = \sum_{i=1}^{n}\mathrm{E}\left[\log\left(1+\frac{\rho_k}{n}\tilde{\lambda}_{k,i}\right)\right] \tag{42}$$

is the link capacity of user k subject to the SWNSF scheduling. When user k is in the low SNR regime, (42) can be approximated by using (32), that is,

$$\tilde{C}_k \simeq \frac{\rho_k}{n}\sum_{i=1}^{n}\mathrm{E}[\tilde{\lambda}_{k,i}] = \rho_k\left[n + \frac{1}{n}\left(\psi(K+1)+\beta-1\right)\right] \quad \text{for small } \rho_k \quad . \tag{43}$$

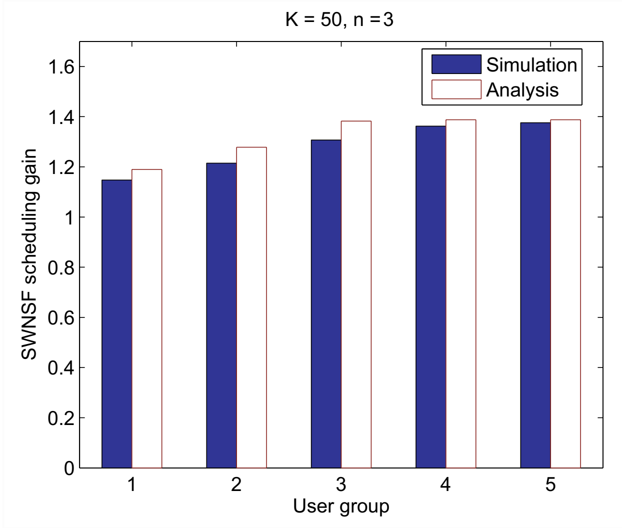

Fig. 4. Capacity improvement with the SWNSF scheduling for the users in different user groups.

When user k is in the high SNR regime, $\tilde{C}_k$ in (42) can be upper bounded by

$$\tilde{C}_k \le n\log\left(\frac{\rho_k}{n}\right) + n\log\left[n + \frac{1}{n}\left(\psi(K+1)+\beta-1\right)\right] \quad \text{for large } \rho_k \quad , \tag{44}$$

where the derivation is in Appendix B.

For comparison, we also express the system capacity with the RR scheduling as

$$\langle C\rangle_{\text{RR}} = \sum_{k=1}^{K} p_k C_k = \frac{1}{K}\sum_{k=1}^{K} C_k \quad , \tag{45}$$

where C_k is the link capacity of user k under the RR scheduling. Again, since the RR algorithm does not alter the statistics of the receive SNR for any target user, C_k in (45) is the same as (6). In [37], a closed-form expression for C_k is given as

$$C_k = e^{n/\rho_k}\sum_{i=0}^{n-1}\sum_{j=0}^{i}\sum_{l=0}^{2j}\left\{\frac{(-1)^l}{2^{2i-l}}\binom{2i-2j}{i-j}\binom{2j}{j}\binom{2j}{l}\sum_{r=0}^{l}E_{r+1}\left(\frac{n}{\rho_k}\right)\right\} \quad , \tag{46}$$

where $E_r(z) = \int_1^\infty e^{-zt}t^{-r}dt$ is the exponential integral function of order r [29]. Next, we evaluate the capacity gain of $\tilde{C}_k$ over C_k when applying the SWNSF scheduling.

E. Numerical Example

Here, we give some numerical examples to demonstrate the capacity benefits brought by the SWNSF scheduling. We divide the total K users equally into five groups. The users in the same group have the same value of average SNR ρ. The values of ρ from the first group to the fifth group are set to [30, 20, 10, 0, −10] dB, respectively. In the following, we apply the low SNR approximation of (43) and the high SNR upper bound of (44) to evaluate the link capacity of the users in groups $\{4, 5\}$ and that in groups $\{1, 2, 3\}$, respectively.

TABLE I
COVERAGE AND CAPACITY ENHANCEMENTS WITH THE SWNSF SCHEDULING FOR $K = 20, 50$ AND $n = 1, 2, 3$

Multiuser MIMO systems		$K = 20$ Relative Cell Radius	$K = 20$ Relative Capacity Gain	$K = 50$ Relative Cell Radius	$K = 50$ Relative Capacity Gain
$n = 1$	SISO	1	1	1	1
$n = 2$	MIMO with RR scheduling	0.71	1.94	0.71	1.94
$n = 2$	MIMO with SWNSF scheduling	1.52	2.74	1.65	2.94
$n = 3$	MIMO with RR scheduling	0.58	2.89	0.58	2.89
$n = 3$	MIMO with SWNSF scheduling	1.24	3.54	1.35	3.72

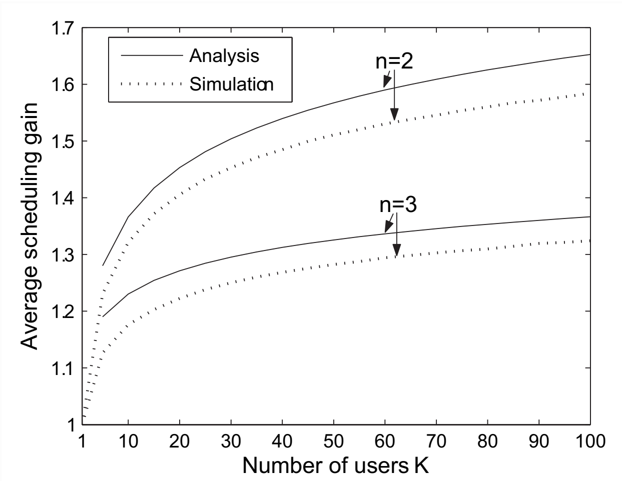

Fig. 5. Average capacity gain resulting from the SWNSF scheduling with different numbers of users in the system.

Figure 4 shows the capacity improvement accomplished by the SWNSF scheduling for the users in different groups. The performance metric is the ratio of the link capacity achieved by the SWNSF scheduling to that by the RR scheduling, i.e. $\tilde{C}_k/C_k$. We assume $K = 50$ and $n = 3$ in this example. As one can see from Fig. 4, the SWNSF scheduling can provide 15% to 38% capacity gain over the RR scheduling for the users in the groups 1 to 5, respectively. This implies that the SWNSF scheduling tends to bring about higher capacity gain for the users with lower ρ.

Figure 5 shows the average capacity gain resulting from the SWNSF scheduling with different numbers of users in the system. The performance metric shown in the y-axis is the average capacity gain defined by $\frac{1}{K}\sum_{k=1}^{K}\frac{\tilde{C}_k}{C_k}$. One can see from Fig. 5 that the capacity gain increases as K increases. Furthermore, the average capacity gain achieved by the SWNSF scheduling diminishes due to the effect of channel damping when a larger number of antenna elements is employed.

VI. Concluding Remarks

Before making conclusions, we present Table I to summarize how the SWNSF scheduling quantitatively enhances the coverage and capacity of multiuser MIMO systems. We normalize both the cell radius and link capacity for the SISO system to unity. The cell radius and link capacity for $K = 20, 50$ and $n = 2, 3$ are shown in Table I in comparison with the SISO case. From the coverage performance aspect, the large number of antennas for the spatial multiplexing MIMO system can result in coverage shrinkage because of link reliability degradation. However, the SWNSF scheduling can virtually extend coverage by improving the receive SNR quality. From the capacity performance aspect, one can see that, as expected, the capacity of the spatial multiplexing MIMO system grows linearly with the number of antennas. Moreover, applying the SWNSF scheduling can further improve the capacity for the MIMO system.

In conclusion, we have demonstrated the advantage of using multiuser diversity to enhance the reduced link quality of the diversity-deficient spatial multiplexing MIMO system. We introduce a fair SWNSF scheduling algorithm which can be applied to a multiuser MIMO system with only low-rate feedback channels. Through tractable eigenvalue analysis, we show that the SWNSF scheduling can enhance the receive SNR of all subchannels for any selected user so that better link reliability (and thus coverage extension) and higher link throughput (and thus system capacity improvement) can be achieved. In the future, it is worth further investigating an optimal scheduling algorithm for the multiuser MIMO system subject to different tradeoffs such as diversity gain, multiplexing gain, feedback requirement and implementation cost.

Appendix A
Proof of Proposition 3

We prove Proposition 3 in this Appendix.

Proof of (i): By applying the variable transformation (29) to (14) and using the result of Proposition 2, the joint PDF of $s_{k,1}, \cdots, s_{k,n}$ can be written as

$$f_{s_{k,1},\cdots,s_{k,n}}(s_1,\cdots,s_n) = ne^{-ns_n}\left\{\frac{1}{n\left[\prod_{i=1}^{n}\Gamma(i)\right]^2}\right.$$
$$\left.\exp\left(-\sum_{i=1}^{n-1}s_i\right)\prod_{i=1}^{n-1}s_i^2\cdot\prod_{\substack{i<j\\ i,j\in\{1,\cdots,n-1\}}}(s_i-s_j)^2\right\}$$
$$= f_{s_{k,n}}(s_n)f_{s_{k,1},\cdots,s_{k,n-1}}(s_1,\cdots,s_{n-1})\ . \quad (47)$$

Thus, for any s_i $(1 \le i \le n-1)$, the joint PDF of s_i and s_n can be derived as

$$f_{s_{k,i},s_{k,n}}(s_i,s_n) = \int_{s_i}^{\infty}\cdots\int_{s_2}^{\infty}\int_0^{s_i}\cdots\int_0^{s_{n-2}} f_{s_{k,1},\cdots,s_{k,n}}(s_1,\cdots,s_n)ds_{n-1}\cdots ds_{i+1}ds_1\cdots ds_{i-1} = f_{s_{k,n}}(s_n)f_{s_{k,i}}(s_i)\ . \quad (48)$$

Thus, $s_{k,i}$ and $\lambda_{k,n}$ are independent.

Proof of (ii): Since the spacing $s_{k,i}$ are independent of $\lambda_{k,n}$, we have $\tilde{\lambda}_{k,i} = s_{k,i} + \tilde{\lambda}_{k,n}$ where $s_{k,i}$ and $\tilde{\lambda}_{k,n}$ are also independent. Thus, it is followed that

$$\phi_{\tilde{\lambda}_{k,i}}(\omega) = \phi_{\tilde{\lambda}_{k,n}}(\omega)\phi_{s_{k,i}}(\omega) = \phi_{\tilde{\lambda}_{k,n}}(\omega)\left[\frac{\phi_{\lambda_{k,i}}(\omega)}{\phi_{\lambda_{k,n}}(\omega)}\right]\ . \quad (49)$$

Note that the Laplace transform $\phi_{\lambda_{k,n}}(\omega) = n/(n+\omega)$ [29] and

$$\phi_{\tilde{\lambda}_{k,n}}(\omega) = \int_0^{\infty} f_{\tilde{\lambda}_{k,n}}(\lambda)e^{-\omega\lambda}d\lambda = Kn\int_0^{\infty}(1-e^{-n\lambda})^{K-1} \cdot e^{-(n+\omega)\lambda}d\lambda = \frac{K!\,\Gamma(1+\omega/n)}{\Gamma(1+K+\omega/n)}\ , \quad (50)$$

where we have used the integral identity [34, eq. 3.312]

$$\int_0^{\infty}\left(1-e^{-x/a}\right)^{b-1}e^{-cx}dx = \frac{a\Gamma(ac)\Gamma(b)}{\Gamma(ac+b)}\ . \quad (51)$$

Combining (50) and (49) yields the result of (30).

Proof of (iii): Since the appropriate derivative of the Laplace transform of a PDF evaluated at its argument $\omega = 0$ gives rise to moments, this proof of (31) is completed by

$$\begin{aligned} \mathrm{E}[\tilde{\lambda}_{k,i}] &= -\left.\frac{d\phi_{\tilde{\lambda}_{k,i}}(\omega)}{d\omega}\right|_{\omega=0} \\ &= -A(0)\left.\frac{d\phi_{\lambda_{k,i}}(\omega)}{d\omega}\right|_{\omega=0} - \phi_{\lambda_{k,i}}(0)\left.\frac{dA(\omega)}{d\omega}\right|_{\omega=0} \\ &= \mathrm{E}[\lambda_{k,i}] + \frac{1}{n}\left[\psi(K+1)+\beta-1\right]\ . \end{aligned} \quad (52)$$

Note that we have utilized $\phi_{\lambda_{k,i}}(0) = 1$ and $d\Gamma(\omega)/d\omega = \Gamma(\omega)\psi(\omega)$ [29, eq. 6.3.1] in the derivation of (52).

Proof of (iv): We first establish the following identity

$$\sum_{i=1}^{n}\mathrm{E}\left[\lambda_{k,i}\right] = \mathrm{E}\left[\mathrm{tr}\left(\mathbf{H}_k\mathbf{H}_k^{\dagger}\right)\right] = \mathrm{E}\left[\|\mathbf{H}_k\|_F^2\right] = n^2 \quad (53)$$

where $\|\mathbf{H}\|_F = \sqrt{\sum_i\sum_j |h_{ij}|^2}$ is the matrix Frobenius norm and $\mathrm{tr}(\cdot)$ is the trace of a matrix. Then (32) is achieved by using (53) and (28)

$$\sum_{i=1}^{n}\mathrm{E}[\tilde{\lambda}_{k,i}] = n\,\Delta_\lambda + \sum_{i=1}^{n}\mathrm{E}[\lambda_{k,i}] = n^2 + n\,\Delta_\lambda\ . \quad (54)$$

APPENDIX B
DERIVATION OF (44)

In this Appendix, we derive the upper bound for the link capacity under the SWNSF scheduling when ρ_k is large. Starting from (42), we have

$$\begin{aligned} \tilde{C}_k &= \mathrm{E}\left[\log\det\left(\mathbf{I} + \frac{\rho_k}{n}\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^{\dagger}\right)\right] \\ &\overset{(a)}{\simeq} \mathrm{E}\left[\log\det\left(\frac{\rho_k}{n}\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^{\dagger}\right)\right] \\ &\overset{(b)}{=} n\log\left(\frac{\rho_k}{n}\right) + \mathrm{E}\left[\log\left(\prod_{i=1}^{n}\tilde{\lambda}_{k,i}\right)\right] \\ &\overset{(c)}{\le} n\log\left(\frac{\rho_k}{n}\right) + n\mathrm{E}\left[\log\left(\frac{\sum_{i=1}^{n}\tilde{\lambda}_{k,i}}{n}\right)\right] \\ &\overset{(d)}{\le} n\log\left(\frac{\rho_k}{n}\right) + n\log\left[n + \frac{1}{n}(\psi(K+1)+\beta-1)\right]\ , \end{aligned} \quad (55)$$

where (a) follows from the large ρ_k approximation, (b) from $\det(\tilde{\mathbf{H}}_k\tilde{\mathbf{H}}_k^{\dagger}) = \prod_{i=1}^{n}\tilde{\lambda}_{k,i}$, (c) from the arithmetic-geometric inequality, and (d) from (40) together with the Jensen's inequality.

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